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Estimation of the consumption function using the Almon Technique

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Estimation of the consumption function using the Almon Technique

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CHAPTER I. INTRODUCTION

The consumption function was introduced into economic analysis by Keynes, who argued that the marginal propensity to consume out of current income is less than unity, and that the average propensity to consume declines as real income rises (8, pp. 96, 97). The assumption of a stable consumption function is crucial to the Keynesian theory. The marginal propensity to consume plays the same role in Keynesian theory as does velocity in the quantity theory of money. If velocity is a constant or a stable function of a few known variables, then the effect of a change in the money supply upon economic activity will be predictable. If the marginal propensity to consume is a constant or a stable function of a few known variables, then the effect of changes in "autonomous" variables such as investment, government spending, and taxes upon the level of economic activity will be predictable.

The assumption that the average propensity to consume declines as real income rises implies a secular tendency for the average propensity to consume to fall. In such a case, non-consumption spending must rise as a fraction of total income, or else total spending would fall short of total output and full employment would be unattainable. This result could be avoided if investment (as a fraction of income) were to rise, but this would involve a rise in the capital-labor ratio. The "secular stagnationists" viewed the latter as unlikely, and believed that increasing government fiscal stimulus would be needed in order to maintain full employment.

The theoretical controversy raised by the Keynesian theory led to various attempts to measure the consumption function. Both the early time series and cross-section studies appeared to confirm Keynes' hypothesis. But estimates of saving since 1899 made by Kuznets showed no tendency for the fraction of income saved to rise despite large increases in income (10, pp. 507-526). The implied average propensity to consume, which, since constant, is then also the marginal propensity to consume, is higher than the marginal propensities to consume estimated in the earlier studies. Further, in the early cross-section studies, the average propensity to consume is about the same at various dates, yet the marginal propensity is less than the average propensity.

This evidence discredited the stagnation theory and suggested that perhaps there is a difference between short- and long-run adjustments of consumption to changes in income. Figure I-1 shows this difference.

This apparent difference between the short run and long run consumption functions led to the development of more complex hypotheses of the consumption function, such as the relative income hypothesis, according to which consumption depends upon the ratio of current income to peak past income (4).

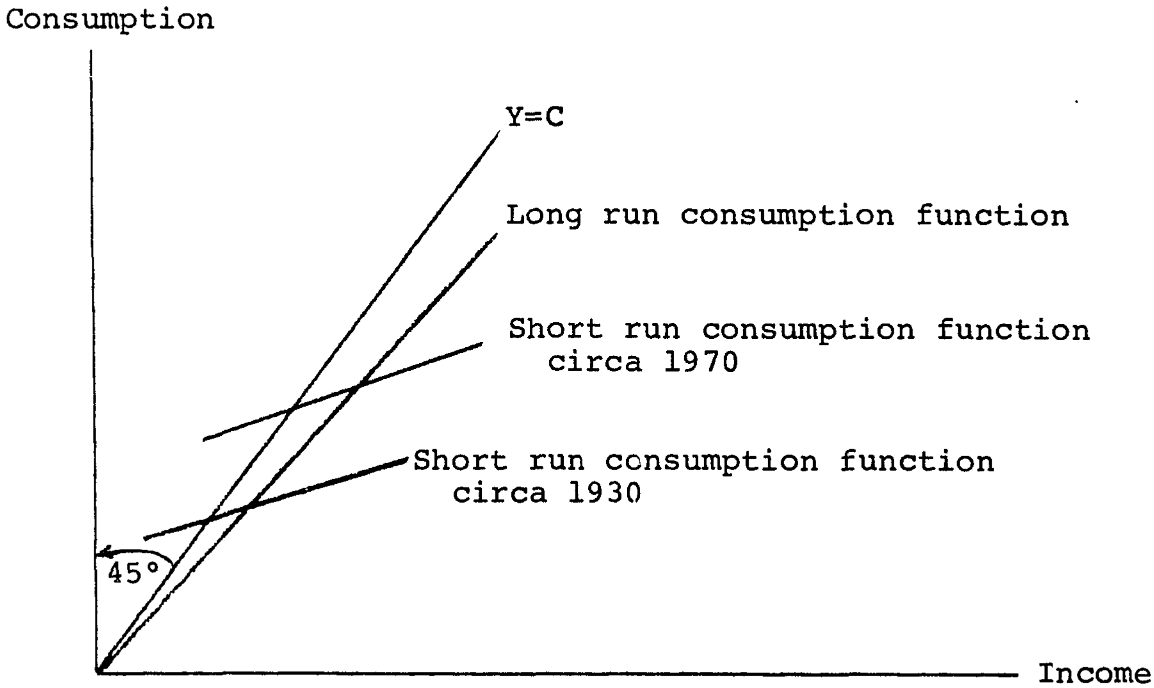

Figure I-1. Short run and long run consumption functions

Another line of criticism resulted from the Keynesian claim that there was no automatic tendency for the economy to move toward a full employment equilibrium through changes in the level of wages and prices. The validity of this proposition depends upon the specification of the consumption function. It is not valid if consumption depends positively on wealth (the "Pigou effect") (7, 12). The same conclusion follows if consumption depends upon the real value of the stock of money (11).

This dissertation is concerned with the estimation of the consumption function according to the permanent income hypothesis, which was proposed by Friedman (5).

Friedman objects to the Keynesian consumption function on the grounds that it relates current consumption to the statistical definition of current income, which is actually current receipts. The usual theoretical definition of income is the amount the consuming unit can consume while keeping its wealth intact. Its wealth is the present discounted value of its current and future receipts. Current income correctly defined (which he labels "permanent income") would then be the interest rate times wealth. Permanent income is thus an expected income concept, and current receipts or measured income can be smaller or larger than permanent income.

> "A similar problem arises about the meaning of 'consumption.' We have been using the term consumption to designate the value of the services that it is planned to consume during the period in question, which, under conditions of certainty, would also equal the value of the services actually consumed. The term is generally used in statistical studies to designate actual expenditures on goods and services. It therefore differs from the value of services it is planned to consume on two counts: first, because of additions to or subtractions from the stock of consumer goods, second, because of divergencies between plans and their realization" (5, p. 11).

Thus "permanent consumption" is planned consumption of services including the use value of consumer durables. Purchases of consumer durables represent saving, since they add to the wealth of the consuming unit. The hypothesis then is that permanent consumption is a function of permanent income and the interest rate. Furthermore, given the interest rate, permanent consumption is a constant fraction of permanent income, under conditions of certainty.

The introduction of uncertainty into the analysis creates a new motive for holding wealth--the need for a reserve in case of emergencies.

> "All forms of wealth are not, however, equally satisfactory as a reserve for emergencies. The major general distinction is between human and nonhuman wealth. In a nonslave society, there is no market in human beings comparable to the market for nonhuman capital. It is in general far easier to borrow on the basis of a tangible physical asset, or a claim to one, than on the basis of future earning power. Accordingly, current consumption may be expected to depend not only on total permanent income and the interest rate, but also on the fraction of permanent income derived from nonhuman wealth, or—what is equivalent for a given interest rate—on the ratio of nonhuman wealth to permanent income. The higher this ratio, the less need there is for an additional reserve, and the higher current consumption may be expected to be" (5, p. 16).

Other variables which would affect any consuming unit's ratio of permanent consumption to permanent income are age, size of family, and education.

The differences between measured and permanent income and consumption are labeled transitory income and consumption, respectively.

In order to make the theory capable of contradiction, Friedman specifies that there should be zero correlation between permanent and transitory income, permanent and transitory consumption, and between transitory income and transitory consumption. If in addition it is assumed that the mean transitory components of income and consumption are zero, the empirical difference between the short-run and long-run marginal propensities to consume can be resolved.

This point can be explained with the aid of Figure I-2. Here, the consumers with measured income y_o have higher than average income. While Friedman assumes zero correlation between permanent and transitory income, the correlation between transitory income and itself is unity. Therefore there is a positive correlation between transitory income and measured income, which is the sum of permanent and transitory income. Thus, on the average, the consumers with income y_o have positive transitory income. However, there is assumed to be no correlation between transitory income and consumption, and mean transitory consumption is assumed to be zero. Thus permanent income for consumers with measured income y_o is less than y_o, say, y_{p_o}. On the average they will consume ky_{p_o}, (where k is the average and marginal propensity to consume out of permanent income) or $y_{p_o}G$ in Figure I-2. In other words, the consumers with measured income y_o consume $y_{p_o}G = y_oF$. But since their measured income is greater than their permanent income, their marginal propensity to consume out of measured income is smaller than their marginal propensity to consume out of permanent income.

Loosely put, the argument is that the short run consumption function is flatter because the high income groups contain,

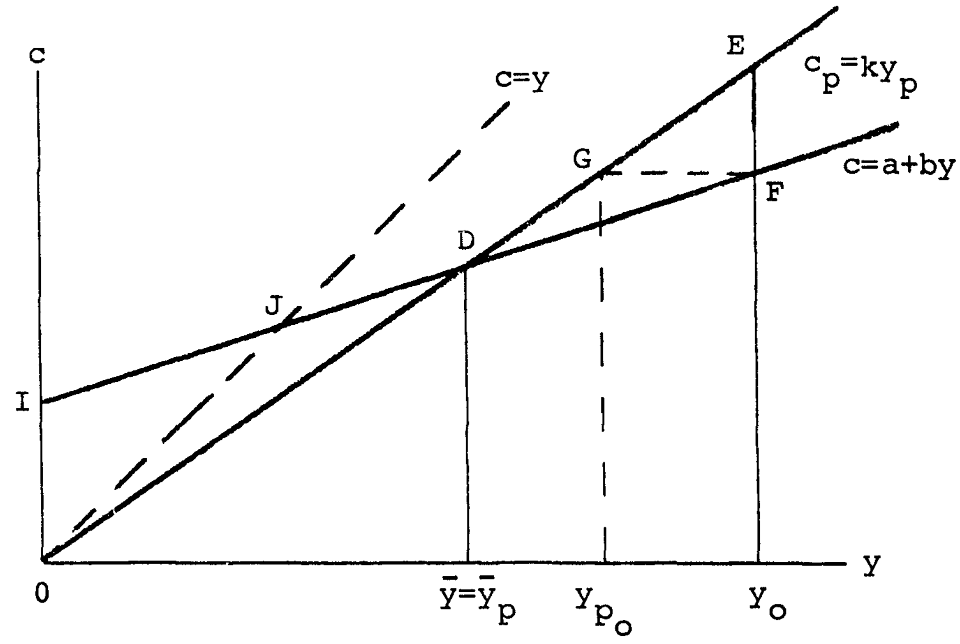

Figure I-2. Hypothetical relation between measured consumption and measured income (this reproduces Figure 3 in (5, p. 34)

on the average, more people whose incomes are temporarily or unusually high than low. Since they base their consumption decisions on their permanent incomes, they will spend less than if they expected these high incomes to be permanent. Of course the opposite argument holds for the low-income groups.

Examining real per capita consumption and disposable income data for 1897-1949, Friedman finds no secular tendency for the average propensity to consume to rise or fall. The most notable variations in this ratio seem to be caused by wars and depressions: the lowest values are recorded in the war years 1917, 1918 and 1942-1945, the highest values in the depressions of 1921 and the 1930's. This behavior is entirely consistent with the permanent income hypothesis since one would expect transitory income to be negative in a depression and positive in a war boom. Also, one would expect transitory consumption to be negative in wartime because of shortages and rationing.

Friedman attributes the long-run constancy of the propensity to consume (k) to offsetting changes in the factors which affect it. The major ones are: (1) the decline in the number of farm families, which would tend to raise k, because farmers, like other entrepreneurs, have lower than average k, (2) the decline in the average size of the family, which would reduce k because large families have higher than average k, (3) the vast increase in government social security and welfare programs might or might not have affected k. No definite conclusion is possible since increases in government pension funds are counted as saving but it is not clear how much the existence of the programs has caused private saving to fall.

Apparently operating on the assumption that the variables affecting the size of k have offset each other over the long run, Friedman sets out to estimate consumption as a linear function of permanent income. Since permanent income cannot be observed, it is assumed that consumers behave as if they estimate permanent income as a weighted average of current and past income.

"One alternative is to construct a weighted average of longer series of years, allowing both the weights and the number of years to be determined by the data; the weights, by multiple correlation, the number of years, by adding years until an additional year produces no significant increase in the correlation. Unpleasantly complex in theory, this alternative also has the statistical defect that it uses up an undue number of degrees of freedom in application. But it does indicate a direction along which to proceed (5, p. 142).

Instead he chooses to impose a weighting pattern upon the data. Given that previous studies had indicated a relatively heavy weight on current income and less weight on income in the past year, he settles upon an exponentially decaying weight structure. Permanent income at time T is defined as $Y_p^*(T) = \beta \int_{-\infty}^{T} e^{(\beta-\alpha)(t-T)} y^*(t)dt.$[1] $y^*(t)$ is per capita personal disposable income in 1929 prices at year t. β is an adjustment coefficient which must be estimated. α is introduced because of the secular rise in income. Since past income is usually less than current income, a weighted average of past incomes will tend to be less than current income, making the estimate of the marginal propensity to consume out of permanent income "too high." α is the long run average rate of growth of real per capita disposable income, which is taken to be .02 per year. This procedure has the effect of cumulating past income at 2% per year.

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[1](5, p. 144). See Appendix A for the derivation of this formula.

The function fitted to the aggregate data was $c^*(T) = k^*y_p^*(T) + e_T$, where $c^*(T)$ is per capita consumption in 1929 prices at time T and k^* is the estimate of the marginal and average propensity to consume out of permanent income.[2] As noted, the value of β has to be estimated. This was done by selecting various values of β differing by .05, calculating the $y_p^*(T)$ for each, and then regressing $c^*(T)$ on each set of $y_p^*(T)$. The value of β selected was the one used in the regression that had the highest R^2.[3]

The estimate of k^* was .88. This is the same as the estimate for the highest previous income (Duesenberry (4)) hypothesis. The estimated k^* was .90 for the hypothesis that consumption is a function of current income and previous year's income. The big differences among these hypotheses lie in the weight attached to current income in computing permanent income. According to Friedman, current income gets 33% of the weight, compared with 55% for Duesenberry, 64% according to the current and previous year hypothesis, while consumption is a function of current income alone in the simple Keynesian

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[2] A regression was also run including a constant term. However, the latter was not significant (5, p. 146-147).

[3] The actual calculation was performed by Philip Cagan and the method is described in (2).

consumption function.[4]

The implications of this result for economic theory are by no means trivial. In the simplest Keynesian model: $C_t = bY_t$, $I = I_o$; if $b = 0.8$ the multiplier is 5. But if current income gets only one-third of the weight, the marginal propensity to consume out of current income is (.33)(.8) = .264, and the multiplier is only 1.36. If Friedman's results are correct, then the economy is much more stable (that is, changes in "autonomous" spending produce much smaller changes in Gross National Product) than if the Keynesian consumption function is the correct specification.

It is the contention of this writer that while the permanent income hypothesis may be valid, the statistical method employed may result in a biased estimate of the lag structure.

In an ordinary multiple regression the criterion of maximizing R^2 would lead to the inclusion of statistically insignificant independent variables, since R^2 always rises when an additional variable is included in the regression equation (13, p. 14). Friedman's is a simple regression, but the independent

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[4](5, p. 147). The value of β is given as 0.4 and weights for the 17 years, going backward from the present year, are: .330, .221, .148, .099, .067, .045, .030, .020, .013, .009, .006, .004, .003, .002, .001, .001, .001. Actually, if α=0.02 and β=0.4, the implied weights are: .333, .227, .156, .107, .073, .050, .034, .023, .016, .011, .008, .005, .004, .002, .002, .001, .001. This apparent contradiction is resolved in Appendix A.

variable is a weighted average of past years' incomes. So long as there is correlation, even if insignificant, between income in some past year and current consumption, R^2 may be increased by including that year's income in the weighted average. Thus the lag length is increased. But given the exponentially decaying lag function, a longer lag implies a smaller value of β.[5] This, in turn, implies a smaller weight for current income. Thus, the very low weight for current income reported by Friedman may be due, at least in part, to his statistical procedure.

This suspicion is reinforced when one observes the results of trying a different value of β. If $\beta=0.8$, the weights are 0.555, 0.255, 0.117, 0.054, 0.025, 0.011, 0.005, 0.002, 0.001, 0.001, cutting off the lag, as Friedman does, where a weight does not exceed 0.0005. In this case the estimate of k* is .89 and the R^2 is .938, as compared with .944 when $\beta=0.4$.[6] If a very large change in the weight structure produces a very small change in R^2, it is not at all clear that maximizing R^2 should be the criterion, especially if the use of this criterion might produce a biased estimate.

---

[5]The smaller is β, the smaller is the adjustment in any given year to a discrepancy between measured and actual income, and thus the longer the time needed for complete adjustment. The weight given current income is $[\beta/(\beta-\alpha)][1-e^{-(\beta-\alpha)}]$, which varies directly with β. See Appendix A.

[6]Friedman reports $R^2 = .96$. The difference is likely due to a small difference in the data discussed in Chapter III.

Because of these difficulties, it was decided to try other methods of estimating consumption as a function of current and past income. These methods are ordinary least squares and the Almon Technique. The latter is discussed in Chapter II.

CHAPTER II. THE STATISTICAL METHOD

The Almon Technique

The first use of Lagrangian interpolation polynomials in the estimation of a distributed lag was made by Shirley Almon (1), and the procedure has come to be known as the Almon Method, or Almon Technique.

> "All distributed lag equations state that a dependent variable, Y, is determined by a weighted sum of past values of an independent variable, X:
>
> $$Y_t = \sum_{i=0}^{n-1} w(i)X_{t-i}.$$
>
> If n, the number of relevant values of X is small, as may well be the case for some problems if annual data are involved, and if these successive past observations are not collinear, then the w(i), the weights with which the several present and past values are combined, can be estimated directly by least squares. When n is large, however, or when successive observations are too collinear for this straightforward treatment, as will frequently be the case with quarterly data, it becomes necessary to make some reasonable, restrictive assumption about the pattern of the weights. The point is, of course, to choose an assumption which makes the individual lag coefficients depend on a few parameters, which in turn can be estimated in some reasonably simple way.
>
> The "interpolation distribution" assumes that the w(i) are values at $x=0,\ldots,n-1$ of a polynomial w(x) of degree q+1, q<n, where n is the number of periods over which the distributed lag extends. Its estimation is based on the fact that once q+2 points on the curve are known-- $w(x_o) = b_o$, $w(x_1) = b_1, \ldots, w(x_{q+1}) = b_{q+1}$--all the w(i) can be calculated as linear combinations of these known values..." (1, p. 179).

The following is a description of the Almon Technique in the case of a single lagged independent variable, with the weights constrained to lie on a third-degree polynomial. It is based upon the discussion in (3).

Assume that the w(i) are values of a third degree polynomial in i, that w(n) = 0, and that $\sum_{i=0}^{n} w(i) = 1$.

Thus

$$w(i) = a_o + a_1 i + a_2 i^2 + a_3 i^3 = \sum_{k=0}^{3} a_k i^k. \qquad (2.1)$$

A third-degree polynomial has four coefficients. The coefficients are uniquely determined if the polynomial passes through four specified points $(i_k, w(i_k))$. Choose a third-degree polynomial f(i) such that $f(i_k) = w(i_k)$ for four arbitrarily chosen i_k where k = 1, 2, 3, 4. Let f(i) = $\phi_1(i)w(i_1) + \ldots + \phi_4(i)w(i_4)$.

Since $f(i_k)$ must equal $w(i_k)$, $\phi_k(i)$ must be chosen such that when $i = i_k$, $\phi_k(i_k) = 1$ and all other $\phi(i_k)$ must equal zero. (For example, choose the i_k to be i_1, i_2, i_3, i_4. Then when $i = i_1$, $\phi_1(i_1) = 1$, and $\phi_2(i_1) = \phi_3(i_1) = \phi_4(i_1) = 0$.

A polynomial $\phi_k(i)$ which meets these conditions can be constructed:

$$\phi_k(i) = \left[\prod_{\substack{j=1 \\ i_k \neq i_j}}^{4} (i - i_j) \right] \left[\frac{1}{\prod_{\substack{j=1 \\ j \neq k}}^{4} (i_k - i_j)} \right]$$

The i_k are arbitrarily chosen values of i. The results are not affected by the values chosen. However, for the purpose of simplifying the computations, it may be desirable to constrain the lag length. This can be done by leaving out the last $\phi_k(i)$. In the case of a third-degree polynomial the fourth $\phi_k(i)$ would be omitted, and $w(i_4)$, the weight of the last arbitrarily chosen i, would be constrained to zero.

For example, construct the values of $\phi_k(i)$ for a third degree polynomial, constraining w(t-6) to be zero. Select the four arbitrary i_k = 0, 1, 5, 6.

Then $\phi_1(0) = (0-1)(0-5)(0-6) \div (0-1)(0-5)(0-6) = 1.0$

$\phi_2(0) = (0-0)(0-5)(0-6) \div (1-0)(1-5)(1-6) = 0.0$

$\phi_3(0) = (0-0)(0-5)(0-6) \div (5-0)(5-1)(5-6) = 0.0$

$\phi_1(1) = (1-1)(1-5)(1-6) \div (0-1)(0-5)(0-6) = 0.0$

$\phi_2(1) = (1-0)(1-5)(1-6) \div (1-0)(1-5)(1-6) = 1.0$

$\phi_3(1) = (1-0)(1-1)(1-6) \div (5-0)(5-1)(5-6) = 0.0$

The complete set of values of $\phi_k(i)$ would be

i	$\phi_1(i)$	$\phi_2(i)$	$\phi_3(i)$
0	1.0	0.0	0.0
1	0.0	1.0	0.0
2	-0.4	1.2	0.4
3	-0.4	0.9	0.9
4	-0.2	0.4	1.2
5	0.0	0.0	1.0
6	0.0	0.0	0.0
Sum	0.0	3.5	3.5

Our consumption model is

$$c_t = \alpha + \beta \sum_{i=0}^{n} w(i) y_{t-1} + e_t \qquad (2.3)$$

or

$$c_t = \alpha + \sum_{i=0}^{n} \beta\, w(i) y_{t-i} + e_t. \qquad (2.4)$$

The form of $\beta w(i)$ is unknown and is to be estimated from the data.

This can be done by assuming values of b_k (to be estimated) such that:

$$\beta w(i) = \sum_{k=1}^{3} \phi_k(i) b_k. \qquad (2.5)$$

Or equivalently, $w(i) = \frac{1}{\beta} \sum_{k=1}^{3} \phi_k(i) b_k$

In Figure II-1, i_1, i_2, i_3, and i_4 are the arbitrarily selected values of i. $w(i_4)$ is constrained to zero by assumption.

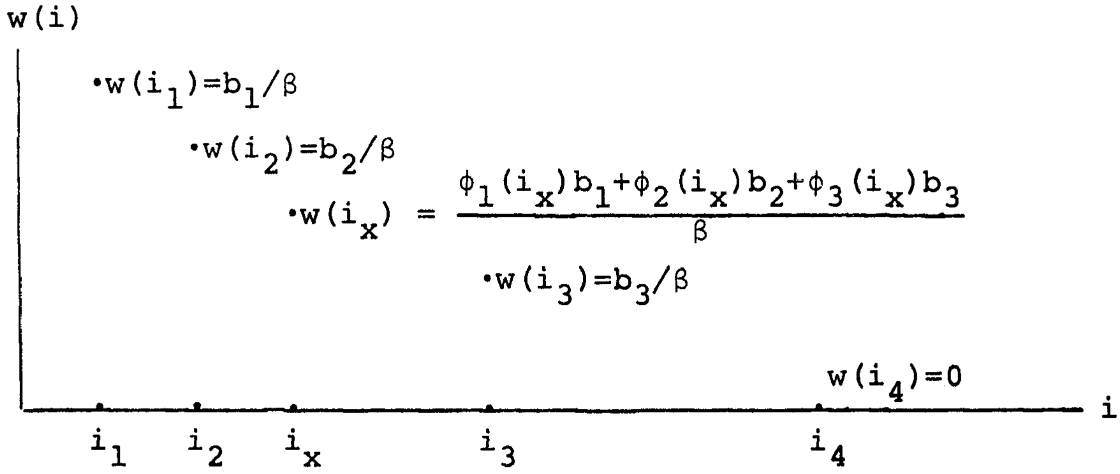

Figure II-1. Weights calculated by the Almon Technique

The values of $\phi_k(i)$ are constants independent of the data. The w(i) depend on β and the b_k, which must be estimated from the data.

Since

$$c_t = \alpha + \sum_{i=0}^{3} \beta\, w(i)\, y_{t-i} + e_t \qquad (2.6)$$

$$c_t = \alpha + \sum_{i=0}^{n} \left[\sum_{k=1}^{3} \phi_k(i) b_k \right] y_{t-i} + e_t \qquad (2.7)$$

$$c_t = \alpha + \sum_{i=0}^{n} \sum_{k=1}^{3} \phi_k(i) b_k\, y_{t-i} + e_t \qquad (2.8)$$

$$c_t = \alpha + \sum_{k=1}^{3} b_k \sum_{i=0}^{n} \phi_k(i) y_{t-i} + e_t \qquad (2.9)$$

The "Almon variables" are defined as:

$$A_t^k = \sum_{i=0}^{n} \phi_k(i)\, y_{t-i}$$

The procedure is to compute the Almon variables from the $\phi_k(i)$ constants and the values of current and past income, and then to estimate the equation:

$$c_t + \alpha_o + b_1 A_t^1 + b_2 A_t^2 + b_3 A_t^3 + e_t, \qquad (2.10)$$

to get the estimates, $\hat{b}_k$ of the b_k.

The restriction that the weights sum to one has been imposed, so that:

$$\beta = \beta \sum_{i=0}^{n} w(i)$$

Since by (2.5) $\beta w(i) = \sum_{k=1}^{3} \phi_k(i) b_k$ then

$$\beta \sum_{i=0}^{n} w(i) = \beta = \sum_{i=0}^{n} \sum_{k=1}^{3} \phi_k(i) b_k \tag{2.11}$$

$$\beta = \sum_{k=1}^{3} \sum_{i=0}^{n} \phi_k(i) b_k \tag{2.12}$$

$$\beta = \sum_{k=1}^{3} b_k \sum_{i=0}^{n} \phi_k(i). \tag{2.13}$$

The estimate $\hat{\beta}$ of β is then

$$\hat{\beta} = \sum_{k=1}^{3} \hat{b}_k \sum_{i=0}^{n} \phi_k(i) = \hat{b}_1 \sum_{i=0}^{n} \phi_1(i) + \hat{b}_2 \sum_{i=0}^{n} \phi_2(i) + \hat{b}_k \sum_{i=0}^{n} \phi_3(i). \tag{2.14}$$

Since $\hat{\beta}\hat{w}(i) = \sum_{k=1}^{3} \phi_k(i) \hat{b}_k$ then

$$\hat{w}(i) = \frac{1}{\hat{\beta}} \sum_{k=1}^{3} \phi_k(i) \hat{b}_k \tag{2.15}$$

If the null hypothesis is that $w(i) = 0$, the appropriate test for significance is the t-test: t_α, df $= \hat{w}(i)/\text{St.E.}w(i)$. The estimate of

$$\text{St.E. } w(i) \text{ is } \sqrt{\frac{\text{var } y}{\hat{\beta}^2}} = \frac{1}{\hat{\beta}} \sqrt{\sum_{k=1}^{3} \phi_k^2(i)\,\text{Var}(\hat{b}_k)}$$

where the $\text{Var}(\hat{b}_k)$ are the estimated variances of the regression coefficients.

It should be noted that the Almon Technique "gains something" (saves degrees of freedom) only when the lag length is greater than the degree of the polynomial. When the two are equal, the Almon Technique reduces to ordinary least squares. For example, assume a third-degree polynomial and a three period lag. Then the $\phi_k(i)$ constants will be

i	$\phi_1(i)$	$\phi_2(i)$	$\phi_3(i)$
0	1.0	0.0	0.0
1	0.0	1.0	0.0
2	0.0	0.0	1.0
3	0.0	0.0	0.0
Sum	1.0	1.0	1.0

The Almon variables, $A_t^k = \sum_{i=0}^{2} \phi_k(i)\, y_{t-i}$ will be:

$$A_t^1 = 1.0y_t + 0.0y_{t-1} + 0.0y_{t-2} = y_t$$

$$A_t^2 = 0.0y_t + 1.0y_{t-1} + 0.0y_{t-2} = y_{t-1}$$

$$A_t^3 = 0.0y_t + 0.0y_{t-1} + 1.0y_{t-2} = y_{t-2}$$

The estimate $\hat{\beta}$ of β is

$$\hat{b}_1 \sum_{i=0}^{2} \phi_1(i) + \hat{b}_2 \sum_{i=0}^{2} \phi_2(i) + \hat{b}_3 \sum_{i=0}^{2} \phi_3(i) = \hat{b}_1 + \hat{b}_2 + \hat{b}_3.$$

The estimate of $w(i)$, $\hat{w}(i) = \frac{1}{\hat{\beta}} \sum_{k=1}^{3} \phi_k(i) b_k$.

Then $\hat{w}(0) = \hat{b}_1/\hat{\beta}$, $\hat{w}(1) = \hat{b}_2/\hat{\beta}$, $\hat{w}(2) = \hat{b}_3/\hat{\beta}$.

The standard error of $w(i)$ is $\frac{1}{\beta} \sqrt{\sum_{k=1}^{3} \phi_k^2(i) \mathrm{Var}(b_k)}$.

So the estimate of the standard error of $w(0)$, St.E. $\hat{w}(0) = \frac{1}{\hat{\beta}} \sqrt{\mathrm{Var}(\hat{b}_1)}$ = St.E. $(\hat{b}_1)/\hat{\beta}$, and St.E. $\hat{w}(1)$ = St.E. $(\hat{b}_2)/\hat{\beta}$, and and St.E. $\hat{w}(2)$ = St.E. $(\hat{b}_3)/\hat{\beta}$.

Serial Correlation

Serial correlation of the residuals is a common problem in time series studies. Its presence may result in biased estimates of the coefficients.[7] Dickson (3, pp. 43-47) uses an approximation to the technique of generalized least squares to remove this problem. The autocorrelation coefficient ρ is estimated $e_{t+1} = \rho e_t + u_t$. Then the variables are transformed as follows: $c'_{t+1} = c_{t+1} - \rho c_t$, and $y'_{t+1} = y_{t+1} - \rho y_t$.

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[7]However, the presence of serial correlation does not necessarily imply biased estimates (13, pp. 67-77).

The Almon variables are then recalculated using the y' values, and then c' is regressed on the new Almon variables. If the Durbin-Watson statistic still indicates significant serial correlation, this procedure is repeated. Dickson reported that it usually took two, and sometimes three iterations to eliminate the serial correlation (3, p. 65).

He also reported that after this procedure was performed, the lag structures became "more smoothed and economically justifiable," in the sense that the weights no longer formed a U-shaped pattern (3, p. 44).

Determining the Length of the Lag

In order to determine the proper lag length, Dickson (3, p. 68) started with a ten-quarter lag. If the weight for the tenth quarter was significant at the 0.05 level for a one-tailed test, longer lag lengths were tried. If the weight for the tenth quarter was not significant, the lag was shortened until the last weight was significant.

Peter Schmidt and Roger N. Waud (14) emphasize that the use of Almon Technique involves the imposition of restrictions which, if not true, lead to biased and inconsistent estimates and invalid tests. They demonstrate that the choice of the lag length and the degree of the polynomial can seriously affect the estimates.

They recommend trying all possible combinations of lag lengths, selecting the one which minimizes the standard error of regression (14, p. 13). This recommendation would apply in the case of more than one lagged independent variable, since the estimates of the lag of one variable will be affected by the lag specified for the others. Dickson would seem to be vulnerable here. On the other hand, Dickson states, "At an earlier date, when computer time seemed plentiful and with a different form of program than the one described in Chapter IV, all possible combinations of the lag lengths from 6 to 12 quarters were tried for all lagged variables. This costly experience proved that the method described above achieved the same results in a much less expensive fashion" (3, p. 68).

Schmidt and Waud recommend using a polynomial of degree no less than four, and using ordinary least squares to check for the existence of a lag, and to obtain information on reasonable lag lengths and degrees of the polynomial (14, p. 13).

The procedure used in this study was to use ordinary least squares, adjusting for serial correlation, for lag lengths up to five years. Estimates were then obtained using the Almon Technique. These necessarily involved using polynomials of degree less than four, since the number of years in the lag is so small.

CHAPTER III. THE DATA

In order that the results of using ordinary least squares or the Almon Technique may be compared with Friedman's results, it is necessary that the data be the same or nearly so. Friedman used "real disposable income per capita and real consumption per capita based on Goldsmith's savings estimates" (5, p. 145). Goldsmith constructed estimates of aggregate personal savings in current values for 1897-1949, and estimates of disposable personal income in current values and in 1929 prices (6, vol. I, p. 345, vol. III, pp. 428-9).

From the two income series one can obtain an implicit price deflator which can then be used to put saving in terms of 1929 prices. By subtracting this series from personal disposable income in 1929 prices one obtains consumption in 1929 prices.

The next step is to get the data into per capita terms. Friedman does not report the population series used. This writer used the series "Total Population Residing in the United States in thousands as of July 1" (15, p. 7).

The resulting estimates of per capita consumption and disposable income in 1929 prices are listed in Appendix B.

One source of difference in the data results from the fact that Friedman included rough extrapolations of consumption and income for 1950 and 1951 (5, p. 145). Since these are not available, they were not used in this study.

Another difference results from Goldsmith's treatment of consumer durables. In his savings study expenditures on consumer durables are treated as saving. On the other hand, Goldsmith's income data are based upon Department of Commerce data which treat expenditures on consumer durables as current outlay.

> "They therefore do not include any allowances for depreciation on the stock on consumer durable goods. However, if consumer durable goods were treated as capital assets in the product and income series, an allowance would also have to be entered for their imputed use value, similar to that now made on account of owner-occupied dwellings. While these two allowances have the tendency to offset each other to a large extent, at least for longer periods of time, the difference in treatment between the estimates of saving and those of national product and income introduces a discrepancy which may occasionally be of importance.
>
> There are thus some conceptual discrepancies--and several minor ones which have been passed over--between the estimates of national product and national and personal income brought together here and estimates of saving developed by the Saving Study. They are, however, sufficiently small not to impair the comparability between the two series. In particular the differences appear to be not important enough to prevent, or even to limit, the use of these estimates of product and income as the independent variable in a statistical analysis and explanation of the behavior of saving, saving being regarded as the dependent variable."[8]

Friedman states that he included the use value in consumption and income; he does not explicitly state that he subtracted the depreciation of consumer durables (5, p. 116).

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[8](6, vol. III, p. 425). See Appendix B for rough estimates of the use value of consumer durables.

This writer is inclined to believe that he did. This is because the estimates of the various propensities to consume using Goldsmith's unadjusted income data are almost the same as those obtained by Friedman.

Friedman reports the following estimates for 1897-1949: average disposable income per capita (1929 prices), $578; average propensity to consume, 0.88; marginal propensity to consume, 0.70 (5, Table 12, line 14, p. 126). This writer obtained $581, 0.88, and 0.70, respectively.

The next step was to attempt to duplicate the results Friedman obtained using the exponentially decaying weighted average.

> ". . . the final function was fitted to data for 1905-51. . . . In the final computation 17 terms were retained in computing expected income . . . The use of this number of terms for the earlier period made it necessary to extend the data back in time. This was done by extrapolating the 1897 [income] figure backward along an exponential growth trend rising at the rate of 2 per cent per year . . . . The war years, 1917, 1918, 1942 through 1945, were excluded on the grounds that special circumstances of those years made it absurd to use a formula like (5.15) to estimate permanent income and that the consumption data had abnormal transitory elements. For similar reasons, in computing permanent income in postwar years, the actual measured income in the war years was replaced by expected income in the last prewar year (1916 and 1941 respectively) plus 2 percent per year to allow for secular growth" (5, pp. 145-146).

These procedures were followed. Friedman regresses consumption on permanent income when the latter is computed with $\alpha=0.02$ (secular trend) and $\beta=0.4$ (adjustment coefficient).

When a constant term is included, its estimate is -4.0 (not significant) and R^2=0.96. Supressing the constant term, the estimated ratio of permanent consumption to permanent income is 0.88 (5, pp. 146-147).

Using the weights implied by Friedman's function, this writer obtained a constant term of -9.3 (also not significant) and an R^2 of 0.944. When the constant term was supressed, the estimate of the ratio of permanent consumption to permanent income was 0.883.

On the basis of these results it was concluded that the data differences are insignificant.

In the estimations by ordinary least squares and the Almon Technique the war years were also omitted. However, Friedman's procedure for computing expected income in the postwar years could not be followed. In effect, he assumes a weight structure and then computes permanent income. In ordinary least squares and the Almon Technique the weights are not assumed, rather, they are estimated from the data. The simplest available solution to this problem was used, namely, for the purpose of estimating consumption as a function of current and past income, the war years were treated no differently from other years (except that, as stated above, consumption in the six war years was excluded from the data).

No attempt was made to include an adjustment for secular growth of income. This writer believes that consumers probably do expect such growth and adjust their spending accordingly. But it was felt that this should be inferred from the data, not imposed upon them. Accordingly, if consumers do adjust for secular income growth, the marginal propensity to consume should be greater the longer the length of the lag. The object of this investigation, however, is the lag structure and not the marginal propensity to consume.

Finally, for all estimates, consumption data for 1905-1949 (except the war years) were used, whereas Friedman included data for 1950 and 1951.

CHAPTER IV. THE RESULTS

The following estimations of consumption as a function of current and past income were made: ordinary least squares using lag lengths of three to six years, the Almon Technique using a third-degree polynomial and a four-year lag, and the Almon Technique using a second-degree polynomial and lag lengths of three and four years.

In each case the initial regression yielded a U-shaped weighting pattern: current income received most of the weight, the immediately prior years received low or even negative weights, and the year farthest in the past received a larger weight. As an example, the following are the results of the first ordinary least squares regression, using a four-year lag.

Table IV-1. Initial regression results

Weight	t-value
w(t) = .707	5.88
w(t-1) = .062	0.34
w(t-2) = .028	0.15
w(t-3) = .203	1.63

The appearance of such an <u>a priori</u> unreasonable result may lead the investigator to suspect some problem with the data, such as multicollinearity, that is, a fixed relation between the independent variables. This suspicion might be reinforced by the observation of high simple correlations between the independent variables, which are likely to occur in a lag model. The recommended solution would probably be the use of some sort of weighted average of the independent variables.

However, it is difficult to know whether or not multicollinearity is present. In particular, high simple correlations between the independent variables are not a sufficient condition for multicollinearity (13, p. 48).

When two variables are collinear, the introduction of the second will cause the standard errors of the coefficients to rise "beyond reasonable limits" (13, p. 49). This was not observed to happen with the data used in this study. It is sometimes said that one can detect multicollinearity by regressing the independent variables in reverse order. If one obtains different estimates of the coefficients, multicollinearity is present. This was tried. The coefficients were identical to several digits.

"...the problem of multicollinearity, except in the sense of a fixed relation between the independent variables as in the example of right and left shoes, does not usually arise in large samples" (13, p. 50). "Although researchers show a growing tendency to blame all econometric problems on this demon, we suggest that it may often be largely a theoretical nightmare rather than an empirical reality" (13, p. 48).

It was concluded that multicollinearity was not the problem.

The Durbin-Watson statistic for this regression was 0.68. This indicates highly significant positive serial correlation of the residuals. The same pattern appeared in all the regressions, whether ordinary least squares or the Almon Technique. The first regression always had highly significant positive serial correlation, with the estimate of the auto-correlation coefficient in the neighborhood of 0.7. When the first iteration was performed, a Durbin-Watson of around 2.7 indicated indeterminately significant negative serial correlation. The estimate of the autocorrelation coefficient was between -0.2 and -0.3. A second iteration resulted in a Durbin-Watson very near to 2.0. In the process, the estimated weights became much more "reasonable."

The final results for the four-year lag are shown below.

Table IV-2. Results of the second iteration

Weight		t-value
w(t) = .591		6.79
w(t-1) = .194		1.76
w(t-2) = .140		1.27
w(t-3) = .075		0.83
Intercept = 7.25		0.45
β = .885	D.W. = 2.08	
$\bar{R}^2$ = .807	Standard Deviation = 19.95	

The ordinary least squares estimates using a five-year lag are shown in Table IV-3 below. The weight for (t-3) is negative, while the weight for (t-4) is positive and significant at the .05 level for a one-tail test. Because of this odd result a six-year lag was estimated. The results are also shown in Table IV-3.

Table IV-3. Final results for lag lengths of five and six years

Weight	t-value	Weight	t-value
w(t) = .521	6.22	w(t) = .505	5.90
w(t-1) = .207	1.98	w(t-1) = .202	1.92
w(t-2) = .152	1.44	w(t-2) = .147	1.39
w(t-3) =-.058	-0.52	w(t-3) =-.050	-0.44
w(t-4) = .178	1.90	w(t-4) = .107	0.90
		w(t-5) = .089	0.97
Intercept = 0.07	0.005	Intercept = 0.71	0.04
β = .921	D.W. = 2.15	β = .917	D.W. = 2.10
S.D. = 19.31	$\bar{R}^2$ = .819	S.D. = 19.13	$\bar{R}^2$ = .838

On the basis of the results of the six-year lag it was concluded that the significance of w(t-4) in the five-year lag was probably due to some quirk in the data.

However, $\bar{R}^2$ does rise and the standard error of the regression falls as the lag length is increased from four to six years. This creates something of a dilemma. If maximization of $\bar{R}^2$ is the criterion, the lag length should be extended, even though the added variables are not significant at the .05 level. On the other hand, if only the years having significant weights are to be included, then $\bar{R}^2$ will not be maximized. The choice of criterion is necessarily arbitrary, as is the choice of the .05 level of significance. This writer is inclined to feel that maximizing $\bar{R}^2$ is more appropriate if the purpose of the model is prediction. But if the purpose is to establish the length of the lag (as it is here) then only the years which have significant weights should be included. It was concluded that a five- or six-year lag could be justified by these estimates.

Table IV-4 presents the results using lags of three and four years for ordinary least squares, the Almon Technique with a second-degree polynomial, and the Almon Technique with a third-degree polynomial.

In this study the length of the lag was determined by the criterion of significance in a one-tail t-test at the .05 level. Using this criterion, all of the estimates indicate that only current income and income in the two previous years have a significant effect on consumption in the current year.

The intercept term was not significant in any of the estimates, and was smaller the longer the lag. This is entirely consistent with the permanent income hypothesis.

Table IV-4. Final estimates of the lag structure of the consumption function

Ordinary least squares		Almon Technique			
		Second-degree polynomial		Third-degree polynomial	
Weight	t-value	Weight	t-value		
w(t) = .606	6.83	w(t) = .555	6.97		
w(t-1) = .196	1.75	w(t-1) = .315	7.53		
w(t-2) = .198	2.19	w(t-2) = .130	2.62		
Intercept = 11.38	0.74	Intercept = 12.68	0.85		
β = .866	D.W. = 2.02	β = .856	D.W. = 2.04		
S.D. = 19.86	$\bar{R}^2$ = .808	S.D. = 20.30	$\bar{R}^2$ = .768		
Weight	t-value	Weight	t-value	Weight	t-value
w(t) = .591	6.79	w(t) = .530	8.04	w(t) = .582	7.17
w(t-1) = .194	1.76	w(t-1) = .300	11.80	w(t-1) = .216	2.78
w(t-2) = .140	1.27	w(t-2) = .135	2.81	w(t-2) = .109	1.56
w(t-3) = .075	0.83	w(t-3) = .035	0.84	w(t-3) = .093	1.39
Intercept = 7.25	0.45	Intercept = 9.15	0.59	Intercept = 7.43	0.46
β = .885	D.W. = 2.08	β = .877	D.W. = 2.05	β = .885	D.W. = 2.05
S.D. = 19.95	$\bar{R}^2$ = .807	S.D. = 19.85	$\bar{R}^2$ = .787	S.D. = 19.62	$\bar{R}^2$ = .813

The estimate of the marginal propensity to consume (β)[9] rose somewhat as the lag was lengthened. This is to be expected because of the rising secular trend of income. This result is consistent with Friedman's interpretation that consumers behave as if they take secular income growth into account when making consumption decisions.

Friedman's estimate of the marginal propensity to consume out of permanent income was 0.88. His method included forcing the constant term in the regression to zero and applying a two percent per year growth factor to past income. Since neither of these adjustments were used in this study, the estimates of β are not directly comparable. However, the estimates obtained here can hardly be said to differ markedly from Friedman's.[10]

With the four-year lag, ordinary least squares and the Almon Technique with a third-degree polynomial yield the same estimate of β. The estimates of the weights are very similar. One possible conclusion is that if there is any problem with the data, such as multicollinearity, both estimates have been

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[9] In this chapter, β is the marginal propensity to consume out of permanent income. Friedman symbolized this concept by k^*, and used β to represent the adjustment coefficient in the formula for permanent income.

[10] When the mean is forced to zero, using the ordinary least squares and a three year lag, the estimate of β is .916. When, in addition, the income data are adjusted by Friedman's method to take into account secular growth of 2% per year, the estimate of β is .905.

affected equally by it. A more reasonable conclusion is that there is no problem with the data, and that the use of the Almon Technique is superfluous in this case. When a second-degree polynomial was employed, the estimate of β was slightly lower than the estimate obtained by using ordinary least squares or the Almon Technique with the third-degree polynomial. Also, there was a substantial difference in the weights. These differences must be due to the use of the second-degree polynomial. This result is consistent with the argument of Schmidt and Waud that the Almon Technique can produce biased estimates if the degree of the polynomial is too small.

For these reasons, this writer considers the ordinary least squares estimate with a three-year lag to be the best. According to this estimate, consumers behave as if they place 60.6% of the weight on current income when they estimate permanent income. While current income gets less weight in the other regressions, it always gets at least half the weight, much more than the one-third reported by Friedman.

If this estimate is the correct one, the marginal propensity to consume out of the current year's income is (.606)(.866) = .525, whereas it would be (.333)(.88) = .293 according to Friedman's estimate. In the simplest Keynesian model, the multipliers, 1/(1-b), would be 2.105 and 1.239, respectively, implying that the economy is substantially less stable than Friedman's estimate would suggest.

CHAPTER V. SUMMARY AND CONCLUSIONS

Economic theory has increasingly accepted expectations of future values or events as determinants of current behavior. Since the future is not directly observable, testing such hypotheses requires that some proxy for the future variable in question be found. A common procedure is to assume that economic units behave as if they estimate the future value of the variable by some sort of average of current and past values of the variable, which are observable.

One of the more prominent examples of this procedure is Friedman's permanent income theory of the consumption function. In the simplified form actually estimated, the theory assumes that households plan to consume a constant fraction of their expected or permanent income. Permanent income is the interest rate multiplied by wealth, which is in turn the present discounted value of all expected future income. Since future income is not observable, Friedman assumed that households behave as if they estimate permanent income from past and current income.

In testing the hypothesis, Friedman rejected on statistical grounds the procedure of simply regressing current consumption on current and past income. Instead, he constrained the estimate of permanent income to be an exponentially decaying average of current and past income adjusted for secular growth. Different estimates of the permanent income series were obtained by varying the adjustment coefficient (exponential rate of decay), and the consumption series was regressed on each. Of the several permanent income series, the one producing the highest value of R^2 in the regression was selected as the best. This was an average of current income and incomes of the sixteen previous years, with current income getting one-third of the weight. An implication for economic theory is that the value of the multiplier is much smaller, and the economy much more stable, than if consumption is a function of current income only.

One difficulty with this procedure is that it may be biased toward an unduly long lag. This is because the inclusion of an additional year in the weighted average may raise R^2 even though the correlation between that year's income and current consumption is not statistically significant. Since the weights are constrained to decay exponentially, the longer is the lag the smaller must be the weight assigned to current income. In this particular case R^2 is not very sensitive to changes in the length of the lag. When the lag was shortened from seventeen years to ten (and the weight given current income was raised from one-third to .55) R^2 fell by less than .01. Such a small difference in R^2 reinforces the suspicion that this procedure may result in the assignment of too small a weight to current income.

Consequently, in this study consumption was regressed on current and past income using ordinary least squares and the Almon Technique. The initial regressions all resulted in an unreasonable U-shaped weight structure, which might be blamed on multicollinearity. There was no evidence that multicollinearity was present, but there was highly significant serial correlation. The latter problem can be removed by applying an iterative version of generalized least squares. When this was done, the lag structure became more "reasonable", that is, the further in the past, the smaller the weight.

The choice of the criterion to be used to decide the length of the lag is arbitrary. This study used the criterion of significance in a one-tail t-test at the .05 level. On this basis, consumption is a function of current income and income in the previous two years only. In the regression judged to be the best, current income received 60.6% of the weight, and in no regression did it receive less than 50%. Assuming these results to be correct, the size of the multiplier in a Keynesian model, and the associated instability of the economy, are greater than Friedman's estimate would suggest. However, the multiplier is smaller and the economy more stable than if current consumption were a function of current income alone.

Using a four-year lag, there was very little difference between the results obtained by using ordinary least squares and those obtained by using the Almon Technique with a third-degree polynomial. This is hardly surprising, since the requirement that the weights in a four-year lag lie on a third-degree polynomial is not a very severe constraint. But the Almon Technique with a second-degree polynomial gave substantially different estimates of the weights in the four-year lag. These results are consistent with two criticisms of the Almon Technique. (1) The estimates may be biased if the degree of the polynomial is too small. (2) Except in cases where the use of ordinary least squares would entail the sacrifice of too many degrees of freedom, there is little to be gained by constraining the lag structure.

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APPENDIX A. A NOTE ON A DISCREPANCY IN FRIEDMAN's A THEORY OF THE CONSUMPTION FUNCTION

The following quotation outlines the method by which Friedman obtained his formula for permanent income. "For this purpose, tentatively regard y_p^* as the 'expected' or predicted value of current measured income. Suppose this expected value is revised over time at a rate that is proportional to the difference between expected and actual income, or

$$\frac{dy_p^*}{dT} = \beta[y^*(T) - y_p^*(T)]. \qquad (5.14)$$

"The solution of this differential equation with suitable initial conditions to make the constant term zero, is

$$y_p^*(T) = \beta\int_{-\infty}^{T} e^{\beta(t-T)} y^*(t)dt, \qquad (5.15)$$

or the estimate stated earlier.

"One obvious defect of this approach is that it does not allow for predicted secular growth. Being an average of earlier observations, the estimated y_p^* is necessarily between the lowest and the highest, so that this method of estimation applied to a steadily growing series yields estimated values systematically below the observed values. To allow for this, we can suppose y_p^* to be estimated in two parts: first, a trend value which is taken to grow at a constant percentage rate, and second, a weighted average of adjusted deviations of past values from the trend, the adjustment being made to allow for the trend change itself, and thus to put all deviations at the same level as the present deviation. This would give:

$$y_p^*(T) = Y_0 e^{\alpha T} + \beta \int_{-\infty}^{T} e^{\beta(t-T)} [y^*(t) - y_0 e^{\alpha t}] e^{\alpha(T-t)} dt, \quad (5.16)$$

where α is the estimated rate of growth and y_0, the value of income at the time taken as zero. This expression reduces to the much simpler form:

$$y_p^*(T) = \beta \int_{-\infty}^{T} e^{(\beta-\alpha)(t-T)} y^*(t) dt, \quad (5.17)$$

and this is the form that we shall use. If we combine (5.17) with our basic consumption Equation (2.10), and recall that measured consumption on the average equals permanent consumption for any given value of measured income, we have as a consumption function to be fitted to aggregate data:

$$c^*(T) = k^* \beta \int_{-\infty}^{T} e^{(\beta-\alpha)(t-T)} y^*(t) dt" \quad (5.18)$$

(5, pp. 143-144).

This equation has three parameters, β, α, and k^*, but only $(\beta-\alpha)$ and k^* can be determined by the fitting process. Therefore it was necessary to choose an arbitrary value for one of the parameters (5, p. 144). "The value of α was taken as .02 on the basis of the secular rate of growth of c*" (5, p. 146). c* is a real per capita consumption.

The first step in the fitting process was to select a value of β. Given this value, permanent income was calculated from formula (5.17). Consumption was regressed on permanent income to obtain the estimate of k*, the ratio of permanent consumption to permanent income. Then another value of β was selected, and the same procedure was applied. Of all the estimates, the one with the highest value of R^2 was the one selected (5, p. 145).

This result was obtained by setting $\beta=0.4$. The resulting weights for income, starting with the current year and moving backwards, are listed as: .330, .221, .148, .099, .067, .045, .030, .020, .013, .009, .006, .004, .003, .002, .001, .001, .001. The estimate of k* was 0.88. The sum of the weights is 1.000.

These are not in fact the weights that result when $\alpha=0$ and $\beta=.4$. The formula for permanent income is:

$$y_p^* = \beta \int_{-\infty}^{T} e^{(\beta-\alpha)(t-T)} y(t)dt.$$

Integrating,

$$y_p^* = \left(\frac{\beta}{\beta-\alpha}\right) e^{(\beta-\alpha)(t-T)} y(t)\Big|_{T-1}^{T} + \left(\frac{\beta}{\beta-\alpha}\right) e^{(\beta-\alpha)(t-T)} y(t)\Big|_{T-2}^{T-1} + \ldots$$

Since y(t) is a step function having a constant value throughout the year,

$$y_p^* = \left(\frac{\beta}{\beta-\alpha}\right) e^{(\beta-\alpha)(t-T)}\Big|_{T-1}^{T} y(T) + \left(\frac{\beta}{\beta-\alpha}\right) e^{(\beta-\alpha)(t-T)}\Big|_{T-2}^{T-1} y(T-1) + \ldots$$

Thus the weight applied to the t^{th} year's income is $(\frac{\beta}{\beta-\alpha})$ $[e^{(\beta-\alpha)(t-T)} - e^{(\beta-\alpha)(t-1-T)}]$, and the sum of the weights is $\beta \int_{-\infty}^{T} e^{(\beta-\alpha)(t-T)} dt = \frac{\beta}{\beta-\alpha}$. Clearly, the sum of the weights cannot be unity unless $\alpha=0$. The weights listed by Friedman on page 147 are in fact those obtained by setting $\alpha=0$. If $\alpha=.02$, the weights sum to approximately 1.053. Rounding to three significant figures, and discarding weights below .0005, the weights (starting with the current year and proceeding into the past) are: .333, .227, .156, .107, .073, .050, .034, .023, .016, .011, .008, .005, .004, .002, .002, .001, .001.

As a check, permanent income was computed using the above weights and those reported by Friedman, and real per capita consumption was regressed on both permanent income series. With the weights as reported by Friedman, the estimate of the marginal propensity to consume was 0.939. Using the weights calculated above, the estimate of the marginal propensity to consume was .875. Friedman reports an estimate of .88, but this was obtained by forcing the constant term to zero (5, pp. 146-147). Following the same procedure, and using the weights calculated above, the estimate of the marginal propensity to consume was .879.

I wrote to Friedman, asking him to clarify these points. He replied, "My recollection, however, is that the reconciliation between the weight summing to one and the use of $\alpha=.02$ is that the α was applied to adjust the basic data (i.e., real income) first and then the weights listed were applied to it. That would give precisely and logically the same result as applying the weights which you list as summing to 1.05. There is certainly no doubt that the weights should sum to 1.05 if the α and β adjustments are both made at the same time."[11]

An adjustment for α can be made in the following manner: recall that permanent income is defined as $y_p^* = \beta\int_{-\infty}^{T} e^{(\beta-\alpha)(t-T)} y^*(t)dt$. Let $y'(t) = y^*(t)e^{-\alpha(t-T)}$. Then $y_p^* = \beta\int_{-\infty}^{T} e^{\beta(t-T)} y'(t)dt$, and in this case the weights will sum to one. The current year's income would be multiplied by $\int_{T-1}^{T} e^{-\alpha(t-T)}dt = \frac{1}{-\alpha}(e-e^{\alpha}) = 1.01$.[12] Preceding years' incomes would be multiplied by $\frac{1}{-\alpha}(e^{-\alpha(t-T)}-e^{-\alpha(t-1-T)})$, a factor which rises as t falls (as we move further into the past). Then each year's income could be multiplied by the weights implied by the various selected values of β. However, it would seem to be simpler merely to calculate the weights directly from Equation (5.17).

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[11]Milton Friedman, Dept. of Econ., University of Chicago, private communication, 1973.

[12]Notice that $\frac{.333}{1.01} = .330$, the weight Friedman reports he applied to current income.

APPENDIX B. TABLES OF DATA

Table B-1. Data

Year	(1)[a] GNP in 1929 prices millions of dollars	(2)[b] GNP in current prices millions of dollars	(3)[c] Reciprocal of implicit price deflator	(4)[d] Total population residing in the U.S. as of July 1, millions
1897	32403	14452	2.2421	72.189
1898	33588	14947	2.2471	73.494
1899	35805	17294	2.0704	74.799
1900	37442	18571	2.0162	76.094
1901	40901	20328	2.0121	77.585
1902	41495	20996	1.9763	79.160
1903	43641	22824	1.9121	80.632
1904	43190	22026	1.9609	82.165
1905	45869	23944	1.9157	83.820
1906	50940	28326	1.7983	85.437
1907	52353	30940	1.6921	87.000
1908	48700	26931	1.8083	88.709
1909	56279	31460	1.7889	90.492
1910	56158	33414	1.6807	92.407
1911	55161	33207	1.6611	93.868
1912	55468	35888	1.5456	95.331
1913	57815	37464	1.5432	97.227
1914	55532	35818	1.5504	99.118
1915	60635	39655	1.5291	100.549
1916	67300	48927	1.3755	101.966
1917	74187	64543	1.1494	103.266
1918	78874	77219	1.0214	103.203
1919	77418	85315	0.9074	104.512
1920	73298	91256	0.8032	106.466
1921	68700	72479	0.9479	108.541
1922	72775	72775	1.0000	110.055
1923	83980	85492	0.9823	111.950
1924	85427	85940	0.9940	114.113
1925	87970	90345	0.9737	115.832
1926	93833	96930	0.9680	117.399
1927	95092	95377	0.9970	119.038

1928	96136	97193	0.9891	120.501
1929	103800	103828	0.9997	121.770
1930	94400	90857	1.0390	123.077
1931	87400	75930	1.1511	124.040
1932	74800	58340	1.2821	124.840
1933	74300	55760	1.3325	125.579
1934	82000	64868	1.2641	126.374
1935	89300	72193	1.2370	127.250
1936	101400	82483	1.2293	128.053
1937	106200	90213	1.1772	128.825
1938	101500	84683	1.1986	129.825
1939	110300	91339	1.2076	130.880
1940	120800	101433	1.1909	131.954
1941	139600	126417	1.1043	133.121
1942	146866	161551	0.9091	133.920
1943	160601	194338	0.8264	134.245
1944	168258	213688	0.7874	132.885
1945	168144	215210	0.7813	132.481
1946	167558	211110	0.7937	140.054
1947	167500	233264	0.7181	143.446
1948	172900	259071	0.6674	146.039
1949	172000	255578	0.6730	148.665

[a]Source: Goldsmith, et al. (6, vol. III, p. 428).

[b]Source: Goldsmith, et al. (6, vol. III, p. 429).

[c]Source: Computed by dividing Column 1 by Column 2.

[d]Source: U.S. Department of Commerce (15, p. 7).

Table B-1 (Continued)

Year	(5)[e] Total personal saving, current prices, millions of dollars	(6)[f] Personal disposable income 1929 prices, millions of dollars	(7)[g] Personal saving per capita, 1929 prices, dollars	(8)[h] Personal disposable income per capita, 1929 prices, dollars	(9)[i] Consumption per capita, 1929 prices, dollars
1897	547	26505	16.98	367.17	350.18
1898	1288	27139	39.38	369.26	329.88
1899	2190	29023	60.61	388.01	237.39
1900	1274	30184	33.75	396.66	362.91
1901	1363	33414	35.34	430.67	395.32
1902	2942	33766	73.45	426.55	353.10
1903	1501	35232	35.59	436.94	401.35
1904	1424	35729	33.98	434.84	400.86
1905	3458	37712	79.03	449.91	370.88
1906	3240	42334	68.19	495.49	427.30
1907	2098	44070	40.80	506.55	465.74
1908	1996	40302	40.68	454.31	413.62
1909	3000	47495	59.30	524.85	465.54
1910	3244	46153	59.00	499.45	440.45
1911	2094	25882	37.05	488.79	451.73
1912	4239	46561	68.72	488.41	419.68
1913	2667	48235	42.33	496.10	453.77
1914	2545	45898	39.80	463.06	324.25
1915	4684	49962	71.23	496.89	425.66
1916	5563	53802	75.04	527.64	452.60
1917	10072	59828	112.10	579.35	467.25
1918	12686	64929	125.55	629.13	503.57
1919	9764	63250	84.77	605.19	520.41
1920	6568	57431	49.55	539.43	489.87
1921	1286	54279	11.23	500.07	488.84
1922	6300	58096	57.24	527.88	470.63
1923	9880	66505	86.69	594.05	507.36
1924	8616	68064	75.05	596.46	521.40
1925	10744	68925	90.31	595.04	504.72
1926	10103	73067	83.30	622.38	539.07

1927	10074	74982	84.37	629.89	545.52
1928	6014	74625	49.36	619.28	569.92
1929	11485	81641	94.29	670.45	576.16
1930	4617	57881	47.41	616.53	569.11
1931	2466	72050	22.88	580.86	557.97
1932	-3273	61336	-33.61	491.31	524.93
1933	-3805	60125	-40.37	478.78	519.15
1934	-954	64947	-9.54	513.92	523.46
1935	2349	71459	22.83	561.56	538.72
1936	5275	80890	50.64	631.69	581.04
1937	7322	83076	66.90	644.87	577.96
1938	3715	77889	34.29	599.95	565.65
1939	6852	84177	63.22	643.16	579.94
1940	8543	89566	77.10	678.76	601.66
1941	13971	104332	115.89	783.73	667.84
1942	33237	104870	225.62	783.07	557.45
1943	36167	107964	222.64	804.23	581.59
1944	39299	113977	232.86	857.71	624.84
1945	36409	116142	214.72	876.66	661.94
1946	22527	124216	127.66	886.91	759.25
1947	20186	118619	101.04	826.92	725.87
1948	26723	122382	122.07	837.69	715.62
1949	22457	112786	101.65	825.92	724.26

[e]Source: Goldsmith, et al. (6, vol. I, p. 345).

[f]Source: Goldsmith, et al.

[g]Source: Computed by multiplying Column 5 by Column 3, and dividing the result by Column 4.

[h]Source: Computed by dividing Column 6 by Column 4.

[i]Source: Computed by subtracting Column 7 from Column 8.

Table B-2. Estimates of the stock of durable consumer goods and the value of services yielded by durable consumer goods[a]

Decade	Stock of consumers' durables, middle of decade[b]	Percent yield	Yield in dollars, per year[b]
1879-88	5,000	4.4	220
1884-93	5,918	4.0	277
1889-98	8,395	3.8	319
1894-03	9,309	3.5	326
1899-08	11,522	3.5	403
1904-13	13,553	3.8	515
1909-18	15,859	4.1	651
1914-23	18,432	4.6	848
1919-28	22,600	4.7	1,062
1924-33	33,755	4.5	1,519
1929-38	28,074	4.0	1,123
1934-43	26,813	3.1	831
1939-48	31,070	2.7	839

[a]Source: Simon Kuznets (9, p. 165).

[b]All dollar figures in millions, 1929 prices.